Queueing Question 2 Solution

Joe loads and unloads a group of 5 machines. All have the same capacity of 10 parts per hour (remember that capacity and cycle times are both averages), and all service times are exponentially distributed. The arrivals of parts from upstream in the process is Poisson and averages 45 parts per hour. After parts leave one of Joe's machines, 1/3 of them pass downstream to Clara. She has a capacity of 20 parts per hour. The remaining parts from Joe's operation go to Sue. Her operation has a capacity of 34 parts per hour.

(a) What is the average waiting time at each queue?

Joe's Queue

$$u = \frac{\lambda}{M \bullet \mu} = \frac{45 \text{ prts/hr}}{5 \text{ machines} * 10 \text{ prts/hr/mach}} = 0.90$$

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} = \frac{.9^{\sqrt{2(5+1)}}}{1-0.9} = 6.94 \text{ parts}$$

$$W_q = \frac{L_q}{\lambda} = \frac{6.94 \text{ parts}}{45 \text{ parts/hr}} * \frac{60 \text{ mins}}{1 \text{ hour}} = 9.26 \text{ mins}$$

Clara's Queue

$$u = \frac{\lambda}{M \bullet \mu} = \frac{(1/3) * 45 \text{ prts/hr}}{1 \text{ machines} * 20 \text{ prts/hr/mach}} = 0.75$$

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} = \frac{.75^{\sqrt{2(1+1)}}}{1-0.75} = 2.25 \text{ parts}$$

$$W_q = \frac{L_q}{\lambda} = \frac{1.27 \text{ parts}}{(1/3) * 45 \text{ parts/hr}} * \frac{60 \text{ mins}}{1 \text{ hour}} = 9.00 \text{ mins}$$

Sue's Queue

$$u = \frac{\lambda}{M \bullet \mu} = \frac{(2/3) * 45 \text{ prts/hr}}{1 \text{ machines} * 34 \text{ prts/hr/mach}} = 0.882$$

$$L_q = \frac{u^{\sqrt{2(M+1)}}}{1-u} = \frac{.882^{\sqrt{2(1+1)}}}{1-0.882} = 6.59 \text{ parts}$$

$$W_q = \frac{L_q}{\lambda} = \frac{6.59 \text{ parts}}{(2/3) * 45 \text{ parts/hr}} * \frac{60 \text{ mins}}{1 \text{ hour}} = 13.2 \text{ mins}$$

(b) What is the average flowtime throughout the entire process?

$$FT_{proc} = FT_{joe} + (1/3)FT_{clara} + (2/3)FT_{sue}$$

$$FT_{proc} = \left[W_{q,joe} + CT_{joe}\right] + (1/3)\left[W_{q,clara} + CT_{clara}\right] + (2/3)\left[W_{q,sue} + CT_{sue}\right]$$

$$FT_{proc} = \left[9.26 + \frac{60}{10}\right] + (1/3)\left[9.00 + \frac{60}{20}\right] + (2/3)\left[13.2 + \frac{60}{34}\right] = 29.2\text{mins}$$

(c) Who should get an additional machine if one wants to minimize flowtime?

If Joe receives an additional server, his utilization will drop from 90% to 45/(6*10) = 75%. Hence, his wait time before his machines will reduce from 9.26 mins to
$W_q = L_q / \lambda + 1/\mu = (1/\lambda) u^{\sqrt{2(M+1)}} / (1-u) = (60/45) 0.75^{\sqrt{2(6+1)}} / (1-0.75) = 1.80$
mins. This will reduce average flowtime by 9.26-1.8= 7.46. mins. If we do the same to Sue's operation, her utilization reduces from wait time reduces from 88.2% to 30/(2*34) = 44.1%. Hence, her wait time will reduce from 14.2 mins down to . $W_q = (60/30) 0.441^{\sqrt{2(2+1)}} / (1-0.441) = 0.482$ mins. Hence, recalling that only 2/3 of parts go through Sue's process, adding a server to Sue's operation will reduce the process flowtime by (13.2-0.482)*(2/3) = 8.48 mins. So Sue should get the machine. Note that since Clara has the same number of servers, but less flow and a smaller wait time, we can safely assume that her improvement will be less than Sue's.

(d) Who should get an additional machine if one wants to maximize capacity?

The answer is Joe, because he has the highest utilization at 90%.